omega academy

فريق أوميغا الأكاديمي يقدم لكم:

اسئلة سنوات فاينل لمادة

مبادئ احصاء

"كل إناء يضيق بما جعل فيه،
إلا وعاء العلم فإنه يتسع"

علي بن أبي طالب -رضي الله عنه-

Q1 :- $X \sim B(10; 0.2)$

$Y \sim \text{poisson}(3)$

$P(X > 3 \cap Y < 2) = \text{--------}$

$\Rightarrow P(X>3)\,P(Y<2)$

$= (1-P(X\leq 3))\,P(Y\leq 1)$

$= (1-.879)(.199)$

$= .024079$

Q2 :-

Score	0	1	2	3	4	5	total
R.F	.05	.40	.15	.10	.20	.10	
$x_i f_i$	0	.4	.3	.3	.8	.5	2.3

$\bar{X} = \text{------}$

$\Rightarrow \bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{2.3}{1} = 2.3$

Q3 :-

X	com.f
10	5
20	14
30	20
40	29
50	35
60	70

$P_{50} = \text{-----}$

$\Rightarrow P_{50} \rightarrow n = 70$

$\frac{X_{np} + X_{np+1}}{2} = \frac{X_{35} + X_{36}}{2} = 55$

Q4 :- 1 black
5 white

$X \sim \text{Geomatric}(P)$, $P = \frac{1}{6}$

$E(X) = \text{------}$

$\Rightarrow E(X) = \frac{1}{P} \Rightarrow E(X) = \frac{1}{\frac{1}{6}} = 6$

Q_5:- $x \sim B(100, .4)$

a- $Var(x) =$ ------

$\Rightarrow = np(1-p)$

$= 100 * .4 * .6 = 24$

b- $\hat{p}$ is normally distributed with mean $\underline{.4}$ and variance $\underline{0.002}$

$\Rightarrow \hat{p} \sim N(p, \frac{p(1-p)}{n})$

$N(.4, 0.002)$

Q_6:- $\bar{x}$ the mean

$n = 50$

$\mu = 40$

$\sigma^2 = 100$

$\bar{x} =$ ------

$\Rightarrow \bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

$= N(40, \frac{100}{50})$

Q_7:- 90^{th} percentile P_{90} of the distribution $N(20, 36)$ --------

$\Rightarrow P_{90}$ of $\bar{x} = P_{90}$ of $N(0,1) * \sigma + \mu$

$= 1.28 * \sqrt{36} + 20$

$= 1.28 * 6 + 20$

$= 27.68$

Q_8:- [redacted]

$x \sim N(10, 36)$

$P(4 < x < 13) =$ -------

$\Rightarrow P(4 < x < 13)$

$= P(\frac{4-10}{6} < Z < \frac{13-10}{6})$

$= P(-1 < Z < .5)$

$= .6915 - .1587$

$= .5328$

Q9:- The 10^{th} percentile $F_{.1}(4,6)$ of the $F_{(4,6)}$ distribution equal ......3.18......

Q10:-

95 % C.I

error = 3

σ previously estimated to be 15

the sample size = ...

$\Rightarrow$ error $= Z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}$

$3 = 1.96 * \frac{15}{\sqrt{n}}$

$\sqrt{n} = 9.8$

$n = 96.04$

Q11:- $n = 100$

Contains 30 with anemia

I) we wish to use this sample for testing whether the Proportion p of people with is more than .20 at

$\alpha = 0.05$

(a) The null and the alternative hypotheses of this test are

$\Rightarrow H_0 : \hat{p} = .3 \quad \Rightarrow \hat{p} = \frac{30}{100} = .30$

$H_1 : \hat{p} > .2$

$\alpha = .05$

(b) test statistic

$$\Rightarrow \frac{\hat{p} - p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{.30 - .2}{\sqrt{\frac{(.3)(.7)}{100}}} = \frac{.1}{0.045} = 2.22$$

& we reject H_1

(C) we will accept the alternative hypothesis if test . st is grater than ......

II) using this sample

a 90 % c.I for p is ------

$\Rightarrow = \hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{m_1}}$

$= .3 \pm 1.96 * .04$

$= .3 \mp 0.0882$

Q 12 :- The number of ways of appointing 3 directors out of 8 condidates = ------

$\Rightarrow 8 \times 7 \times 6 = 336$

Q13 :- A binary number is number whose digits are ether 0 or 1 . The number of possible 6-digit binary number whose third digits is 1 equal ------

$\Rightarrow$ _ _ 1 _ _ _

$= \frac{n!}{(n-r)!} * 1 * \frac{n!}{(n-r)!}$

$$\frac{\frac{2\times1}{1} * 1 \times \frac{3\times2\times1}{?}}{(C_6^{\,2})} = \frac{12}{30}$$

Q14 :- two independent sample from two normally distributied with means μ_1, μ_2 and equal variances

	Sample1	sample2
Sample size	$m=12$	$n=6$
mean	$\bar{x}_1 = 35$	$\bar{x}_2 = 5$
variance	$S_1^2 = 90$	$S_2^2 = 40$

(a) The poded variance S_p^2 of those two samples: -----

$$\Rightarrow S_p^2 = \frac{(m-1)S_1^2 + (n-1)S_2^2}{m+n-2} = \frac{11 * 90 + 5 * 40}{16}$$

$$= 74.375$$

(b) if these two samples are combaind to from a new Sample then the mean of the new sample equal ------

$\Rightarrow$ new mean $= \dfrac{\text{new } \sum x_i}{\text{new size}} = \dfrac{450}{18} = 25$

new $\sum x_i$ $= 35 * 12 + 6 * 5 = 450$

new size $= 12 + 6 = 18$

Q15 :- for Paired sample of 7 pairs , let $D = A - B$ and suppose that $\bar{D} = 3$ and $S_D = 2$

we want to infer about the difference mean μ_D assuming normaly condition satisfid

(a) Give a 90% C.I for μ_D

(b) use this sample for testing $\mu_D > 0$, $\alpha = 0.05$

Q16 :- let $x \sim B(100, 0.5)$ use normal approximation to binomial distribution to approximate :

$P(44 < x \leq 60)$

Key

UNIVERSITY OF JORDAN   MATH 131   DATE: 23/5/2012

DEPT OF MATH   Final EXAM   TIME 2 hrs

Student Name:________________Student Number________

Instructor Name: __________Section Number: _______

Solve all the following questions.

Answers without solution details are not accepted.

Key

Q1) Consider a sample of size n = 100 that has mean 45 and standard deviation 3 .

$n=100$ $\bar{x}=45$ $s=3$

a) Find the interval that contains at least 96 observations

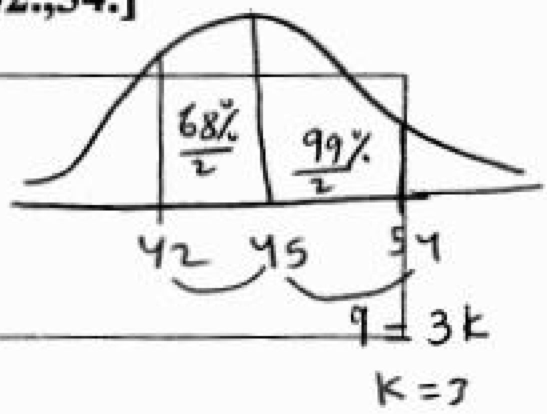
$1-\frac{1}{k^2}=0.96 \Rightarrow k=5$

$(45-5(3),\ 45+5(3))$

$(30,\ 60)$

b) Assuming that the data has a bell−shaped frequency curve

Find the approximate number of observations that may be in the interval [42.,54.]



# of observations = 83.5 ↑

= 84

Q2) Let A and B be two events such that $P(A \mid B) = \frac{1}{4}$, $P(B \mid A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{12}$

a) Find P(A) .

$$P(A) = \frac{P(B \cap A)}{P(B \mid A)} = \frac{1}{12} \cdot \frac{3}{2} = \frac{1}{8}$$

$$P(B) = \frac{P(A \cap B)}{P(A \mid B)} = \frac{1}{12} \cdot 4 = \frac{1}{3}$$

b) Find $P(A \cup B)$.

$$\frac{1}{8} + \frac{1}{3} - \frac{1}{12}$$

$$= 3/8$$

Q3) Let X be discrete random variable with values −1, 0, and 1

Assume that $P(X = 1) = \frac{1}{7}$ and $E(X) = -\frac{1}{2}$

a) Find P(X < 1). $= 6/7$

X	−1	0	1
	c_1	c_2	$\frac{1}{7}$

$$-c_1 + \frac{1}{7} = -\frac{1}{2}$$

$$c_1 = \frac{9}{14}$$

$$c_2 = \frac{3}{14}$$

6/7

b) Find $E(X^2)$.

$Ex^2 = \frac{9}{14} + \frac{1}{7}$

$= \frac{11}{14} = 0.7857$

Q4) Let X be a binomial random variable with parameters n = 130 and $p = \frac{1}{52}$ $\therefore Bi\left(130, \frac{1}{52}\right)$

$EX = \frac{130}{52} = 2.5$

$\sigma^2 = \frac{130}{52}\left(\frac{51}{52}\right)$

$= 2.452$

a) Find $E(X^2)$.

8.7019

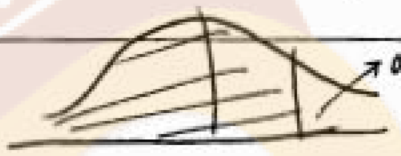
b) Find Poisson approximation for $P(X < 5)$. $= F(4)$

use Poisson table, ~~with~~. Find $F(4)$ from $P(2.5)$

0.891

Q5) Let $\overline{X}$ and S^2 be sample mean and variance of a sample of size n = 16 from normal distribution with mean $\overset{=60}{\mu}$ and variance σ^2

a) Find the 97.5th percentile of $\overline{X}$ if you know that $S^2 = 36$



$P(\overline{X} < c) = 0.975$

$c = \mu_{\bar{x}} + S_{\bar{x}}\, t_{0.025,15}$

$= 60 + \frac{6}{4}(2.131) = 63.1965$

b) Find standard error of $\hat{\mu} = \overline{X}$ if $\sigma^2 = 16$

$\sqrt{\frac{\sigma^2}{n}} = 1$

Q6) Determine the sample size necessary to estimate a proprtion p to within 0.04 with 95.% confidence level

$$0.04 = \sqrt{\frac{\left(\frac{1}{2}\right)^2}{n}}\, 1.96$$

$$0.04 = \frac{\frac{1}{2}(1.96)}{\sqrt{n}}$$

$n \approx \cancel{423}$ 601

Q7) To test the hypothesis that a coin is fair (P({head}=P{tail}=1/2)) against the alternative that it is not fair, toss the coin 122 times and ~~accept~~ reject the hypothesis if the number of heads X ∈ [58,64]

a) Compute probability of type I error of this test

$X \overset{H_0}{\sim} Bi\left(122, \frac{1}{2}\right)$

$Ex = \frac{122}{2} = 61$

$\sigma^2 = 30.5$

$$\alpha = P\left(58 \le X \le 64 \mid P = \tfrac{1}{2}\right)$$

$$= P\left(\frac{57.5 - 61}{\sqrt{30.5}} < Z < \frac{64.5 - 61}{\sqrt{30.5}}\right) = P(-0.63 < Z < 0.63)$$

$$= [0.7357 - 0.2643]$$

$$= \boxed{0.4714} = \cancel{70.5286}$$

b) Compute probability of type II error of this test when P({head})= 0.7

$X \overset{H_1}{\sim} Bi(122, 0.7)$

$Ex = 85.4$

$\sigma^2 = 25.62$

$$\beta = P\left(\frac{57.5 - 85.4}{\sqrt{25.62}} < Z < \frac{64.5 - 85.4}{\sqrt{25.62}}\right)$$

$$P(-5.51 < Z < -4.129)$$

$$= 0$$

Q8) The following table provides information about two paired samples from bivariate normal distribution with mean vector (μ_1 , μ_2)

X	**5**	**6**	**10**	**9**	**8**	**7**
Y	**7**	**12**	**9**	**11**	**10**	**8**
	−2	−6	1	−2	−2	−1

a) Find point estimate for $\mu_1 - \mu_2$

$\bar{d} = -2$

$S_d = 2.2804$

b) Find 90.% confidence interval for $\mu_1 - \mu_2$

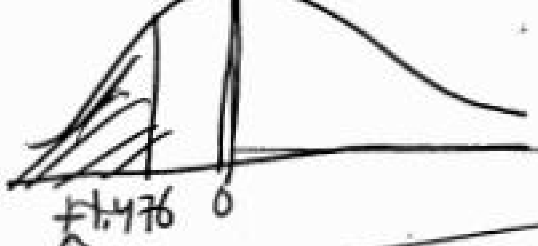
$$\bar{d} \pm \frac{S_d}{\sqrt{n}} t_{0.05,5}$$

$$-2 \pm \frac{2.2804}{\sqrt{6}} (2.015)$$

c) At 0.1 level of significance test H_0 $\mu_1 = \mu_2$ against H_1 $\mu_1 < \mu_2$

Test stat

$$\frac{\bar{d} - \mu_D}{S_d/\sqrt{n}} = \frac{-2\sqrt{6}}{2.2804} = -2.148$$



Reject Ho

Q9) The following table provides information about two independent samples from two independent normal distributions with means μ_1 , μ_2 and variances σ_1^2 , σ_2^2 respectively

	I	II
n	**5**	**7**
$\overline{\mathbf{X}}$	**30**	**28**
$\mathbf{S^2}$	**3**	**6**

a) Find 99.% confidence interval for μ_1

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.005,\,4}$$

$$30 \pm \sqrt{\frac{3}{5}}\ (4.604)$$

b) Find 95% ~~97.5%~~ confidence interval for σ_1^2

$\chi^2_{0.025,\,4} = \cancel{9.48773}$ 11.14330

$\chi^2_{0.975,\,4} = 0.484419$

$$\left(\frac{4(3)}{\cancel{9.48773}\ 11.14330} , \frac{4(3)}{\cancel{0.710721}\ 0.484419} \right)$$

c) Find the pooled variance of the two samples

$$S_p^2 = \frac{4(3) + 6(6)}{10} = 4.8$$

d) Find 90.% confidence interval for $\mu_1 - \mu_2$, $\sigma_1 = \sigma_2$

$$(\bar{x}_1 - \bar{x}_2) \pm \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\; t_{0.05,\,10}$$

$$2 \pm \sqrt{4.8\left(\frac{1}{5} + \frac{1}{7}\right)}\;(1.812)$$

95% 1.283

e) Find ~~97.5%~~ confidence interval for σ_1^2 / σ_2^2 $F_{0.025,4,6} = 6.23$

$$\left(\frac{0.5}{6.23},\; 0.5(9.2)\right) \qquad F_{0.975,4,6} = \frac{1}{F_{0.025,6,4}} = \frac{1}{9.2}$$

Q10) The number of passing and failing male and female students in calculus course are given in the following table

Let p_1 and p_2 be the proportions of passing males and femailes respectively

	Pass	Fail
Male	36	64
Female	41	59

$\hat{p}_1 = 0.36$

$\hat{p}_2 = 0.41$

a) Find the estimated standard error for the estimator of $p_1 - p_2$

$$\sqrt{\frac{\hat{p}_1\hat{q}_1 + \hat{p}_2\hat{q}_2}{100}} = \sqrt{\frac{0.36(0.64)}{100} + \frac{0.41(0.59)}{100}}$$

$$= 0.0687$$

b) Find the point estimate for $p_1 - p_2$

-0.05

c) Find ~~97.5%~~ 95% confidence interval for $p_1 - p_2$

$-0.05 \pm 0.0687(1.96)$

d) Find the pooled proportion of the two samples

$$\bar{p} = \frac{36+41}{200}$$

$$= 0.385$$

0.385

e) Find the rejection region for testing H_0: $p_1 = p_2$ vs. H_1: $p1 \neq p2$, at 0.01 confidence level

or
Test stat
$\frac{0.385}{0.0687} = 5.604$

$(\hat{P}_1 - \hat{P}_2) \pm \sqrt{\bar{P}\bar{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\ Z_{0.05}$

$0.385 \pm 0.0687\ (1.645)$

0.385 ± 0.1130

$(0.272, 0.498)$ $\quad 0 \notin$ C.I. Reject H_0

f) Find the p–value for the testing problem in (e) above

$P\text{-value} = 2\,P(Z > 5.604)$

$= 0$

The University of Jordan
Math 131 Final Exam

Mathematic Department
Date : 20/1/2010
Time : 2 hours

Student name :
Student number :
Course section :

Notes: (1) This exam consists of 33 multiple choice questions .
(2) Carry out all your computations to two decimals.
(3) Mark or circle the correct answer to each question on the following table :

Question 1	a	b	c	d	Question 17	a	b	c	d
Question 2	a	b	c	d	Question 18	a	b	c	d
Question 3	a	b	c	d	Question 19	a	b	c	d
Question 4	a	b	c	d	Question 20	a	b	c	d
Question 5	a	b	c	d	Question 21	a	b	c	d
Question 6	a	b	c	d	Question 22	a	b	c	d
Question 7	a	b	c	d	Question 23	a	b	c	d
Question 8	a	b	c	d	Question 24	a	b	c	d
Question 9	a	b	c	d	Question 25	A	b	c	d
Question 10	a	b	c	d	Question 26	A	b	c	d
Question 11	a	b	c	d	Question 27	a	b	c	d
Question 12	a	b	c	d	Question 28	a	b	c	d
Question 13	a	b	c	d	Question 29	a	b	c	d
Question 14	a	b	c	d	Question 30	a	b	c	d
Question 15	a	b	c	d	Question 31	a	b	c	d
Question 16	a	b	c	d	Question 32	a	b	c	d
					Question 33	a	b	c	d

***For questions (1) – (4)** : Te age distribution of a sample of 30 persons is as follows:

Age. class	10 - 14	15 – 19	20 - 24	25 - 29	30 – 34
Frequency	3	7	10	7	3

1) The sample mean $\overline{X}$ is

a) 23 b) 24.5 c) 22 d) 19.5

2) The proportion of observations greater than 24 is :

a) 1/3 b) 2/3 c) 1/4 d) ¾

3) The 90th percentile is :

a) 27 b) 29.5 c) 28.5 d) 14.5

4) The sample standard deviation S is :

a) 5.63 b) 31.67 c) 5.72 d) 32.76

***For questions (5) –(6)** :

A and B are two events in a random experiment such that P(A)= 0.7 $P(\overline{B})$=0.4
$P(\overline{A} \cup \overline{B})$=0.5

5) $P(A/\overline{B})$=

a) 1/3 b) 0.5 c) 0.2 d) none

6) $P(A \cup \overline{B})$=

a) 0.1 b) 0.9 c) 0.7 d) 0.4

***For questions (7) – (9)** :

A box contains 10 identical cards numbered -1, -1, -1, 0, 0, 1, 1, 2, 3, 3
A number is drawn randomly from the box. Let X=the number drawn .

7) $P(-1 \leq X \leq 1)$=

a) 0.5 b) 0.7 c) 0.3 d) 0.2

8) E(2X-1)=

a) 1 b) 0.4 c) 0.7 d) 0

9)Var(1-2X)=

a) 4.42 b) 8.84 c) 2 d) 8

***For questions (10) – (11)** :

A box contain 4 white balls and 6 black balls. Three balls are drawn together at random.

10) The probability that there are 2 white balls in the 3 balls drawn is :

a) 0.7 b) 0.3 c) 0.288 d) 0.612

11)The expected number of white balls drawn is :

a) 0.4 b) 0.6 c) 1.2 d) 1.6

***For questions (12) – (13)** :
Let X= the number of heads appearing when a balanced coin is tossed 5 times.

12) P(X>1/X<3)=
a) 0.23 b) 1 c) 0.385 d) 0.624

13)Var X=
a) 1.25 b) 1.118 c) 0.5 d) 0.25

***For question (14)** :
The number of typing errors X per page of a book is Poisson random variable with mean 2

14) P(X>4)=
a) 0.947 b) 0.053 c) 0.017 d) 0.983

***For questions (15) – (16)** :
The following table is the joint probability distribution of X and Y

		Y		
		0	1	2
X	0	1/12	1/6	1/12
	1	1/12	1/6	1/12
	2	1/12	1/6	1/12

15) P(X + Y=1)=
a) 1/2 b) 1/4 c) 1/12 d) 1/6

16) P(X≤1/Y=1)=
a) 2/3 b) 1/4 c) 1/3 d) 3/4

***For questions (17) – (19)** :
Let (X, Y) be a bivariate random variable such that EX= 3, EY=5, Var X=4, VarY=9, Cov(X,Y)=-1

17) Corr(3X-2, 4-2Y)=
a) –1/36 b) 1/6 c) –1/6 d) 1/36

18) E(X^2 + 2X + 5)=
a) 24 b) 15 c) 19 d) 20

19) Var(2X-3Y + 4)=
a) 109 b) 7 c) 85 d) 113

***For questions (20) – (24)** :
The grades of an exam are normally distributed with mean 63.6 and variance 25

20) The minimum grade for the top 10% of the grades is
a) 80 b) 90 c) 70 d) 75

21)The proportion p of grades greater than 70 is

a) 0.05 b) 0.9 c) 0.95 d) 0.1

22)Ten grades are randomly selected, use(binomial n=10 and p=0.1) to find the probability that at least two of these ten grades are each greater than 70. This probability is :

a) 0.07 b) 0.736 c) 0.264 d) 0.93

23) Hundred grades are randomly selected, use(binomial n=100 and p=0.1) to find the probability that exactly 10 of these grades are each greater than 70. This probability is:

a) 0.4325 b)0.135 c) 0.5675 d) 0.9108

24)The probability that the average of a sample $\overline{X}$ of 25 grades is less than 65 is :

a) 0.0308 b)0.9192 c) 0.3897 d)0.6103

<u>*For questions (25) – (26)</u> :

Two independent random samples of university students (n_A=100 and n_B=150) were randomly selected to compare two teaching methods (A and B). The number of passing and failing students under each method were as follows :

	Number passing	Number failing
Method A	63	37
Method B	107	34

25)A 98% CI for the passing percentage of method A p_A is

a) (0.625, 0.635) b) (0.52, 0.74)

c) (0.28, 0.46) d) (0.367, 0.373)

26) For testing if there is a difference (i.e. $p_A \neq p_B$) in proportion of passing under the methods A and B, the value of the Z test statistics is :

a) –1.88 b) 2.58 c) -11.83 d) -1.33

<u>*For questions (27) – (30)</u> :

The lifetime (age) in hours of two independent random samples of two types of batteries (I, II) gave the following data

Battery type	Sample size	Sample mean	Sample standard deviation
I	9	90	3
II	10	80	4

Let μ_1, μ_2 be the respective population means of the two batteries, and let σ_1, σ_2 be the respective population standard deviations.

27) A 98% CI for μ_1 is :

a) (87.1, 92.9) b) (86.18, 92.82)

c) (93.03, 90.97) d) (6.1, 11.9)

28) A 98% CI for σ_1 is :

a) (1.09, 17.86) b) (3.28, 53.57)

c) (1.89, 6.61) d) (2.04, 4.23)

29) A 95% CI for $\mu_1 - \mu_2$ is :

a) (-2.34, 22.34) b) (8.42, 11.58)

c) (4.37, 15.63) d) (6.54, 13.46)

30) For testing $H_0 : \sigma_1^2 = \sigma_2^2$ vs. $H_1 : \sigma_1^2 \neq \sigma_2^2$ at $\alpha = 0.10$, we accept H_0 if the statistic S_1^2 / S_2^2 belongs to the interval

a) (0.39, 2.47) b) (0.29, 3.23)

c) (0.31, 3.45) d) (0.40, 2.56)

***For question (31)** :

A researcher wants to determine if a certain medicine reduces blood pressure (BP) of patients. He conducted an experiment and data were as follows :

Patient Number	1	2	3	4	5	6	Mean	Standard deviation
Bp before (X)	70	80	76	74	68	84	75.33	6.02
Bp after (Y)	68	72	58	74	72	74	69.67	5.59
d=X-Y	2	8	18	0	-4	10	5.67	7.25

31) A 99% CI for the mean BP reduction is :

a) (-7.4. 18.74) b) (1.22, 10.12)

c) (-6.26, 17.6) d) (-5.24, 16.58)

***For question 32** :

It is desired to estimate the population proportion p of defective items by a 96% CI.

32) The sample size needed so that the 96% CI has length 0.15 (i.e. error of estimation=0.075) is :

a) 136 b) 34 c) 187 d) 47

***For question (33)** :

It is desired to test H_0 : $\mu=50$ vs. H_1 : $\mu>50$ using the sample mean $\overline{X}$ of a large sample of 100 observation

33) If the value of the Z test statistics is 1.36, the P-value of this test is :

a)0.4131 b) 0.1738 c) 0.2066 d) 0.0869

END OF EXAM